

Sound Absorption and Dispersion in Dilute Polyatomic Gases: A Generalized Kinetic Approach

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Abstract

A generalized kinetic model equation which takes into account the frequency dependence of the thermal conductivity is used to analyze the problem of sound propagation in dilute polyatomic gases. By comparing the theoretical results with some available experimental data we infer that our model equation provides a precise transition between low and high-frequency limits.

Keywords: polyatomic gases, kinetic model equation, frequency-dependent thermal conductivity, sound propagation

1 Introduction

Essentially, there are two different approaches to solve the problem concerning the propagation of plane harmonic waves in gases, namely: macroscopic and microscopic approaches. The macroscopic approach is based on conservation equations of mass, momentum and energy, and on the laws of Navier-Stokes and Fourier. Sound propagation predictions derived from this macroscopic approach are valid as long as the oscillation frequency of the acoustic wave is smaller than the mean molecular collision frequency [1]. In this case, sound dispersion is negligible, while the absorption per wavelength is proportional to the oscillation frequency and can be written as a sum of contributions due to viscous and thermal effects. Different generalizations of the usual macroscopic approach - trying to extend its validity to the high-frequency region - are reported in the literature [2, 3, 4]. Among them, the most popular are those that take into account the frequency dependence of the transport coefficients. However, the sound propagation results derived from this approach give good agreement with the experimental data only for the phase velocity of the sound waves. When the oscillation frequency is comparable to the molecular collision frequency one has no recourse unless to turn to a microscopic approach, i.e., one has to use the Boltzmann equation. One of the earliest attempts to solve the problem of sound propagation in polyatomic gases based on a kinetic equation was made by Hanson, Morse and Sirovich [5]. By employing the method of Sirovich and Thurber for polyatomic kinetic models - which is based on a generalization of the Gross-Jackson procedure for monatomic gases - they derived dispersion relations for the four-moment and seven-moment approximations. Calculations performed by Hanson, Morse and Sirovich for nitrogen and oxygen show that their results are in some agreement with the experimental results of Greenspan [6].

Our purpose in this paper is to study the propagation of sound waves in dilute polyatomic gases using a kinetic model equation which replaces the collision operator of the Boltzmann equation by a single relaxation time term [7] and generalizes the classical hydrodynamic description by considering

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a frequency-dependent thermal conductivity. In particular, our expression for the generalized thermal conductivity follows from the Maxwell-Cattaneo law of heat conduction which can be written as a constitutive equation for the heat flux vector with an exponential memory kernel. By applying the normal mode method to our generalized model equation, it is possible to derive a dispersion relation which can be used to determine the phase velocity and the attenuation coefficient of the acoustic mode. Comparison of the theoretical results with the acoustic measurements performed by Greenspan in oxygen and nitrogen shows that our generalized kinetic model equation for dilute polyatomic gases provides a precise transition between low and high-frequency limits.

We organize the paper as follows: in Section 2 we derive a generalized kinetic model equation for dilute polyatomic gases which takes into account the frequency dependence of the thermal conductivity. The sound wave propagation problem is analyzed in Section 3, while in Section 4 we compare the theoretical results with some available experimental data. Finally, in Section 5, we finish with some concluding remarks.

Cartesian notation for tensors with the usual summation convention is used. Furthermore, angular parentheses around indices denote traceless symmetrization.

2 Generalized Model Equation

In the classical kinetic theory developed by Taxman [8] for dilute polyatomic gases, the one-particle distribution function $f(\mathbf{x}, \mathbf{c}, t, s)$ is defined in such a way that $f(\mathbf{x}, \mathbf{c}, t, s)d\mathbf{c}ds$ gives the number of molecules at position \mathbf{x} and time t with molecular velocities between \mathbf{c} and $\mathbf{c} + d\mathbf{c}$ and with internal degrees of freedom between s and $s + ds$. By neglecting external forces, the one-particle distribution function satisfies the Boltzmann equation [9]

$$\frac{\partial f}{\partial t} + c_i \frac{\partial f}{\partial x_i} = \mathcal{C}(f, f), \quad (1)$$

where $\mathcal{C}(f, f)$ is the Boltzmann collision operator. The collision operator describes the rate of change of the distribution function due to molecular collisions and obeys the following collisional invariant conditions [10]:

$$\int m \mathcal{C}(f, f) d\mathbf{c} ds = 0, \quad (2)$$

$$\int m c_i \mathcal{C}(f, f) d\mathbf{c} ds = 0, \quad (3)$$

$$\int \left(\frac{mc^2}{2} + \mathcal{E}_s \right) \mathcal{C}(f, f) d\mathbf{c} ds = 0, \quad (4)$$

where m is the molecular mass and \mathcal{E}_s denotes the energy associated with the internal degrees of freedom. Conditions (2)-(4) correspond to the conservation of mass, momentum and energy during collisions, respectively.

The mathematical complexity of the collision operator $\mathcal{C}(f, f)$ to analyze time-dependent problems like sound propagation and light-scattering in dilute polyatomic gases is usually avoided by replacing it by a single relaxation-time term of the form

$$\mathcal{C}(f, f) = -\frac{(f - f_r)}{\tau}, \quad (5)$$

where τ is an effective relaxation time and f_r is a reference distribution function which satisfies the main physical properties of the Boltzmann collision operator. Recently, an expression for the reference distribution function was obtained by Marques Jr. [11] by requiring the Chapman-Enskog solution of the kinetic model equation to be consistent with the classical Navier-Stokes-Fourier description. In the usual Navier-Stokes-Fourier theory a macroscopic state of a dilute polyatomic gas is characterized by the fields of mass density

$$\rho = \int m f d\mathbf{c} ds, \quad (6)$$

flow velocity

$$v_i = \rho^{-1} \int m c_i f d\mathbf{c} ds, \quad (7)$$

and temperature

$$T = (\rho c_v)^{-1} \int \left(\frac{m C^2}{2} + \mathcal{E}_s \right) f d\mathbf{c} ds, \quad (8)$$

while the pressure tensor

$$p_{ij} = \int m C_i C_j f d\mathbf{c} ds \quad (9)$$

and the heat flux vector

$$q_i = \int \left(\frac{m C^2}{2} + \mathcal{E}_s \right) C_i f d\mathbf{c} ds \quad (10)$$

are given, respectively, by the following constitutive relations [12]

$$p_{ij} = \left(p - \eta_0 \frac{\partial v_r}{\partial x_r} \right) \delta_{ij} - 2\mu_0 \frac{\partial v_{\langle i}}{\partial x_{j \rangle}} \quad (11)$$

and

$$q_i = -\lambda_0 \frac{\partial T}{\partial x_i}, \quad (12)$$

where c_v is the total specific heat at constant volume, $C_i = c_i - v_i$ is the peculiar velocity, p is the gas pressure, μ_0 is the shear viscosity, η_0 is the bulk viscosity and λ_0 is the thermal conductivity. By taking the effective relaxation time τ equal to the stress relation time $\tau_s = \mu_0/p$, the reference distribution function for dilute polyatomic gases which is compatible with the classical Navier-Stokes-Fourier theory reads

$$\begin{aligned} f_r = f^{(0)} & \left\{ 1 + \tau \left(1 - \frac{f_0}{\gamma} \right) \left[\left(\frac{m C^2}{2kT} - \frac{5}{2} \right) + \left(\frac{\mathcal{E}_s}{kT} - \frac{\mathcal{E}}{kT} \right) \right] \frac{C_i}{T} \frac{\partial T}{\partial x_i} \right. \\ & \left. + \tau (1 - Z_0) \left[\left(\frac{5}{3} - \gamma \right) \left(\frac{m C^2}{2kT} - \frac{3}{2} \right) - (\gamma - 1) \left(\frac{\mathcal{E}_s}{kT} - \frac{\mathcal{E}}{kT} \right) \right] \frac{\partial v_r}{\partial x_r} \right\}, \end{aligned} \quad (13)$$

where

$$f^{(0)} = \frac{\rho}{m} \left(\frac{m}{2\pi kT} \right)^{3/2} \exp \left(-\frac{m C^2}{2kT} \right) \frac{\exp(-\mathcal{E}_s/kT)}{\mathcal{Z}} \quad (14)$$

is the equilibrium distribution function and

$$\mathcal{Z} = \int \exp(-\mathcal{E}_s/kT) ds \quad (15)$$

is the partition function associated with the internal degrees of freedom. Moreover, γ is the specific heat ratio, $f_0 = \lambda_0/\mu_0 c_v$ is the Eucken factor, $Z_0 = (\eta_0/\mu_0)/(5/3 - \gamma)$ is the internal relaxation number (i.e., the mean number of molecular collisions required for the translational and internal degrees of freedom to come to thermal equilibrium [13]) and

$$\mathcal{E} = \frac{1}{\mathcal{Z}} \int \mathcal{E}_s \exp(-\mathcal{E}_s/kT) ds \quad (16)$$

is the mean internal energy.

By analyzing time-dependent problems like sound propagation and light scattering in dilute polyatomic gases, Marques Jr. [11] was able to determine the range of applicability of his kinetic model equation. Comparison of theoretical results with available experimental data in nitrogen, oxygen, carbon dioxide and methane shows that the reference distribution function (13) is valid as long as the external oscillation frequency is smaller than the relaxation frequency for internal and translational degrees of freedom to come to thermal equilibrium.

It is possible to extend the validity of the kinetic model equation to the high-frequency region by considering a generalization of the classical hydrodynamic description. Certainly, the most popular approach that generalizes the hydrodynamic behaviour of simple fluids is based on the Maxwell-Cattaneo law of heat conduction [3]

$$\tau_q \frac{\partial q_i}{\partial t} + q_i = -\lambda_0 \frac{\partial T}{\partial x_i}, \quad (17)$$

where the relaxation time τ_q gives us a measure of the time interval spent by the heat flux vector to achieve a stationary value. It is well known [14, 15] that the Maxwell-Cattaneo equation (17) can be written as

$$q_i = - \int_0^t \frac{\lambda_0}{\tau_q} \exp\left(-\frac{t-t'}{\tau_q}\right) \frac{\partial T(\mathbf{x}, t')}{\partial x_i} dt', \quad (18)$$

i.e., as a constitutive equation with an exponential memory kernel in such a way that a generalized thermal conductivity can be defined as

$$\lambda(t-t') = \frac{\lambda_0}{\tau_q} \exp\left(-\frac{t-t'}{\tau_q}\right). \quad (19)$$

Since sound wave and light scattering solutions are found by Fourier transformation in space and time, we verify from (19) that this generalization introduces a frequency-dependent thermal conductivity

$$\lambda(\omega) = \frac{\lambda_0}{1 + i\omega\tau_q}. \quad (20)$$

Furthermore, based on the work of Zwanzig [4], it is possible to modify the constitutive relation (11) for the pressure tensor to take into account the frequency dependence of the bulk viscosity. Aiming to calculate the spectral distribution of scattered light in a one-component fluid whose molecules have internal degrees of freedom weakly coupled to their translational degrees of freedom by a single relaxation time process, Mountain wrote the pressure tensor as [16]

$$p_{ij} = \left(p - \eta_0 \frac{\partial v_r}{\partial x_r} - \int_0^t \frac{\eta_0}{\tau_*} \exp\left(-\frac{t-t'}{\tau_*}\right) \frac{\partial v_r(\mathbf{x}, t')}{\partial x_r} dt' \right) \delta_{ij} - 2\mu_0 \frac{\partial v_{\langle i}}{\partial x_{j \rangle}}, \quad (21)$$

where τ_* is the relaxation time. Expression (21) shows that in the so-called weak coupling limit the bulk viscosity consists of two parts, a frequency independent one due to translational motions and a frequency dependent one which is related to the exponential decay of the energy in internal modes as a result of interactions with translational modes. Comparison between theory and experiments shows that the generalized hydrodynamical model proposed by Mountain does not always apply since in some fluids the relaxation of the internal degrees of freedom involves more than one relaxation time.

Based on these facts we construct in this work a generalized kinetic model equation for dilute polyatomic gases which takes into account the frequency dependence of the thermal conductivity via the Maxwell-Cattaneo law, but disregards the frequency dependent part of the bulk viscosity. We start by assuming that the reference distribution function is given by the expression

$$f_r = f^{(0)} \left\{ 1 + A \left(\frac{\mathcal{E}_s}{kT} - \frac{\mathcal{E}}{kT} \right) + A_i C_i + A_{rr} \left(\frac{mC^2}{2kT} - \frac{3}{2} \right) + A_{\langle ij \rangle} C_i C_j + A_{irr} \left(\frac{mC^2}{2kT} + \frac{\mathcal{E}_s}{kT} \right) C_i \right\}, \quad (22)$$

where A , A_i , A_{rr} , $A_{\langle ij \rangle}$ and A_{irr} are expansion coefficients that depend on position and time through the basic fields. As described in details by Marques Jr. [11], the application of the Chapman-Enskog method [9] to solve the kinetic model equation allows the determination of the expansion coefficients appearing in the above reference distribution function. Hence, by considering the constitutive relations (11) and (18) we get

$$\left(\frac{5}{3} - \gamma \right) A + (\gamma - 1) A_{rr} = 0,$$

$$A_i + \left(\frac{5}{2} + \frac{\mathcal{E}}{kT} \right) A_{irr} = 0,$$

$$A_{\langle ij \rangle} = 0, \quad (23)$$

$$A_{rr} = \frac{\mu_0}{p} \left(\frac{5}{3} - \gamma \right) (1 - Z_0) \frac{\partial v_r}{\partial x_r},$$

$$A_{irr} = \frac{\mu_0}{pT} \left[\frac{\partial T}{\partial x_i} - \frac{1}{\tau_q} \int_0^t \frac{f_0}{\gamma} \exp \left(-\frac{t-t'}{\tau_q} \right) \frac{\partial T(\mathbf{x}, t')}{\partial x_i} dt' \right].$$

Finally, by combining expressions (1), (5), (22) and (23) we can write our generalized kinetic model equation for dilute polyatomic gases as

$$\begin{aligned} \frac{\partial f}{\partial t} + c_i \frac{\partial f}{\partial x_i} = & -\frac{p}{\mu_0} (f - f^{(0)}) + f^{(0)} (1 - Z_0) \left[\left(\frac{5}{3} - \gamma \right) \left(\frac{mC^2}{2kT} - \frac{3}{2} \right) - (\gamma - 1) \left(\frac{\mathcal{E}_s}{kT} - \frac{\mathcal{E}}{kT} \right) \right] \frac{\partial v_r}{\partial x_r} \\ & + f^{(0)} \left[\left(\frac{mC^2}{2kT} - \frac{5}{2} \right) + \left(\frac{\mathcal{E}_s}{kT} - \frac{\mathcal{E}}{kT} \right) \right] \frac{C_i}{T} \left[\frac{\partial T}{\partial x_i} - \int_0^t \frac{f(t-t')}{\gamma} \frac{\partial T(\mathbf{x}, t')}{\partial x_i} dt' \right], \end{aligned} \quad (24)$$

where

$$f(t-t') = \frac{f_0}{\tau_q} \exp \left(-\frac{t-t'}{\tau_q} \right) \quad (25)$$

is the generalized Eucken factor.

Closing this section, we call attention to the fact that the application of the above generalized kinetic model equation to study time-dependent problems in dilute polyatomic gases only requires the specification of the ratio of the specific heats γ , the Eucken factor f_0 and the internal relaxation number Z_0 .

3 Absorption and Dispersion of Sound

As an application of the generalized kinetic model equation derived in the previous section, we study the problem concerning the propagation of a plane harmonic wave in dilute polyatomic gases. To simplify this study we consider the sound propagation process in the linear regime near equilibrium and assume that the sound wave moves along the x -axis. Hence, we write the distribution function as

$$f = f_0 \{1 + \psi \exp [i(\kappa x - \omega t)]\}, \quad (26)$$

where f_0 is the absolute equilibrium distribution function, $\kappa = \omega/v + i\alpha$ is the complex wavenumber, ω is angular oscillation frequency, v is the phase velocity, α is the attenuation coefficient and $\psi = \psi(\mathbf{c})$ is a function that depends only on the molecular velocity. In sound propagation problems the complex wavenumber κ is determined as a function of the oscillation frequency ω via solution of the dispersion relation (for details see [11, 17, 18])

$$\det (\mathcal{R}\mathbf{M}^{(0)} - i(\kappa v_0/\omega)(\mathbf{I} - \mathbf{M}^{(1)})) = 0, \quad (27)$$

where $\mathcal{R} = p_0/\mu_0\omega$ is the rarefaction parameter, $v_0 = \sqrt{2kT_0/m}$ is the equilibrium thermal velocity and \mathbf{I} is the identity matrix. Moreover, $\mathbf{M}^{(0)}$ and $\mathbf{M}^{(1)}$ are 3×3 matrices whose elements - given in the Appendix - depend on the dimensionless parameter $z = (\omega/\kappa v_0)(1 + i\mathcal{R})$ and on the plasma dispersion function [19]

$$W(z) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} \frac{\exp(-t^2)}{t - z} dt. \quad (28)$$

The acoustic solution of the dispersion relation (27) in the low-frequency limit can be determined by expanding the dimensionless wave number $\kappa v_0/\omega$ in powers of $1/\mathcal{R}$. By retaining terms up to first order we obtain

$$\begin{aligned} \frac{\kappa c_0}{\omega} &= 1 + \frac{i}{2\gamma\mathcal{R}} \left[\frac{4}{3} + \left(\frac{5}{3} - \gamma \right) Z_0 + \frac{\gamma - 1}{\gamma} f_0 \right] \\ &= 1 + i \frac{\omega}{2\gamma p_0} \left(\frac{4}{3} \mu_0 + \eta_0 + \frac{\gamma - 1}{\gamma} \frac{\lambda_0}{c_v} \right). \end{aligned} \quad (29)$$

We verify from expression (29) that the dispersion of sound is negligible in the low-frequency region, while the sound absorption per wavelength is proportional to the sound oscillation frequency and can be written as a sum of viscous and thermal effects. Note that an additional contribution coming from internal relaxation appears in the absorption coefficient of polyatomic gases, a fact that allows us to determine the bulk viscosity by comparing the theoretical results with available experimental data.

4 Comparison with Experiments

In order to compare the sound propagation results derived from our generalized kinetic model equation (24) with the acoustic measurements performed by Greenspan [6] on nitrogen and oxygen we shall use the following material parameters:

Gas	γ	f_0	Z_0
N ₂	1.40	1.97	2.5
O ₂	1.39	1.95	1.3

The values of the specific heat ratio γ and the Eucken factor f_0 were obtained by using the values of c_v , c_p , μ_0 and λ_0 given in the CRC Handbook of Chemistry and Physics [20], while the values of the internal relaxation number Z_0 were obtained by using expression (29) to fit the sound absorption experiments of Greenspan in the low-frequency limit.

In Fig. 1 and Fig. 2 the attenuation factor $\alpha c_0/\omega$ and the reciprocal speed ratio c_0/v are shown on a double logarithmic scale as a function of the rarefaction parameter $\mathcal{R} = p_0/\mu_0\omega$ for nitrogen and oxygen, respectively. The solid line represents the theoretical sound propagation results derived from our generalized kinetic model equation, while the dashed line represents the theoretical results derived from the usual macroscopic (hydrodynamic) approach based on the laws of Navier-Stokes and Fourier. The open circles are the experimental data of Greenspan for sound wave absorption (lower curve) and dispersion (upper curve).

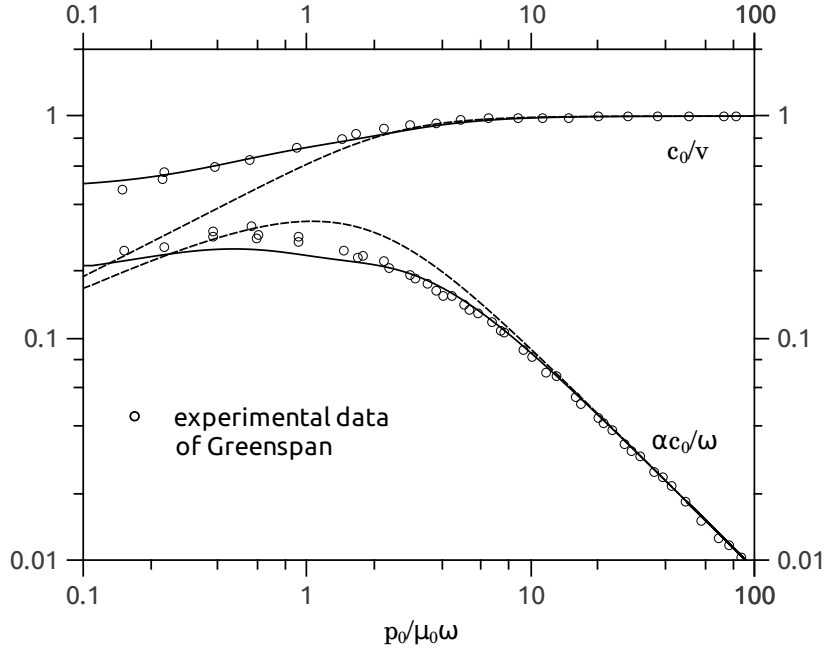


Figure 1: Attenuation factor $\alpha c_0/\omega$ and reciprocal speed ratio c_0/v as a function of the rarefaction parameter $\mathcal{R} = p_0/\mu_0\omega$ for nitrogen at 300K. The predictions of our generalized kinetic model equation (—) are compared with the experimental data of Greenspan and the results of the usual macroscopic approach (---).

We verify from Figs. 1 and 2 that in the low-frequency region ($\mathcal{R} \gg 1$) the theoretical sound propagation results derived from our generalized kinetic model equation are in complete agreement with the experimental data. Moreover, the predictions for sound wave absorption and dispersion are exactly the same as obtained by the macroscopic approach. Since we required the Chapman-Enskog solution of our model equation to be consistent with the usual macroscopic description, this result is in complete agreement with our expectations. In the transition region ($1 < \mathcal{R} < 10$) we note that the kinetic theory proposed in this paper yields better results than the usual macroscopic approach. Whereas the latter predicts a value for the attenuation factor which is about 23% greater than the experimental value, the predictions derived from our generalized kinetic model equation are in good agreement with the absorption data up to a value of the rarefaction parameter of approximately 2. For smaller values of the rarefaction parameter, i.e., in the high-frequency region we observe that

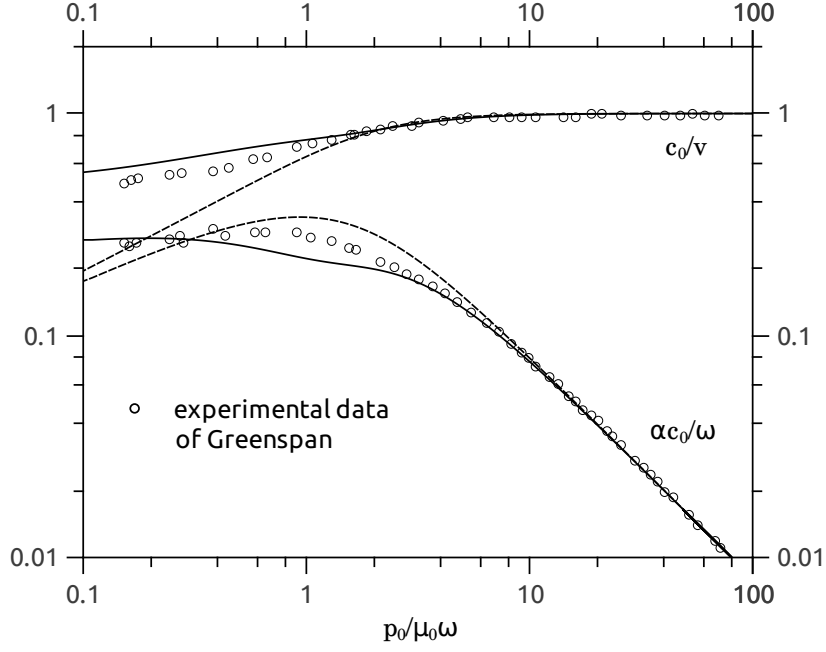


Figure 2: Attenuation factor $\alpha c_0/\omega$ and reciprocal speed ratio c_0/v as a function of the rarefaction parameter $\mathcal{R} = p_0/\mu_0\omega$ for oxygen at 300 K. The predictions of our generalized kinetic model equation (—) are compared with the experimental data of Greenspan and the results of the usual macroscopic approach (---).

the usual macroscopic approach yields a qualitatively correct result for the absorption curve but fails to describe the dispersion data. In contrast, the theoretical results obtained from our generalized kinetic model equation for small \mathcal{R} are in good agreement with both, especially with the sound wave dispersion curve of nitrogen. Further, we can conclude from Figs. 1 and 2 that the agreement between theory and experiment in the high-frequency limit is better for nitrogen than for oxygen, i.e., for gases with higher values of Z_0 which can be explained in terms of the internal relaxation time $\tau = \mu_0 Z_0/p$. A higher value of Z_0 means that the thermal relaxation process between translational and internal degrees of freedom runs more slowly and, therefore, has a greater influence on the propagating sound wave which is appropriately described by our generalized kinetic model equation.

The comparison presented in Figs. 1 and 2 indicates that there are still opportunities for further enhancements in the kinetic theory, such as including the complete frequency dependence of all transport coefficients or considering non-linearities in the model equation, but all in all, we want to remark that the kinetic theory is well suited to describe sound propagation in dilute polyatomic gases over a wide range of frequencies and provides a precise transition between low and high-frequency limits. Whereas the macroscopic approach becomes unable to describe the acoustic measurements in the high-frequency region, the kinetic model equation derived in this work still provides a qualitatively correct description of the experimental data, even for values of the rarefaction parameter smaller than the unity.

5 Conclusion and Outlook

In the present work, we are concerned with the time-dependent problem of sound propagation in dilute polyatomic gases with classical internal degrees of freedom. Since the macroscopic (hydrodynamic) approach fails to describe the experimental data in the high-frequency region one needs to turn to a microscopic (kinetic) approach based on the Boltzmann equation. Assuming that the Boltzmann collision operator can be replaced by a single relaxation-time term in order to eliminate its mathematical complexity, we are able to derive a generalized kinetic model equation for dilute polyatomic gases which explicitly takes into account the frequency dependence of the thermal conductivity. By comparing the theoretical sound propagation results derived from our generalized kinetic model equation with the experimental data, we observe that the kinetic theory is well suited to describe sound propagation in dilute polyatomic gases over a wide range of frequencies. Whereas the macroscopic approach becomes unable to describe the acoustic measurements in the high-frequency region, the generalized kinetic model equation proposed in this work still provides a qualitatively correct description of the experimental data, even for values of the rarefaction parameter smaller than the unity. The presented comparison also indicates that there are still opportunities for further enhancements in the kinetic theory. One of the next steps for future work on this topic would be to include the complete frequency dependence of all transport coefficients into the kinetic model equation.

Finally, we want to close this section by remarking that for the application of our generalized kinetic model equation to a specific sound propagation problem no further information about the potential energy of interaction between the gas molecules is required because the acoustic properties are completely characterized by three parameters, namely the ratio of the specific heats γ , the Eucken factor f_0 and the internal relaxation number Z_0 . Values of these parameters can easily be found in the literature or fitted to experimental data in the low-frequency limit.

Appendix

The non-zero elements of the matrices $\mathbf{M}^{(0)}$ and $\mathbf{M}^{(1)}$ read

$$\mathbf{M}_{11}^{(0)} = W(z), \quad \mathbf{M}_{12}^{(0)} = 2A(z), \quad \mathbf{M}_{13}^{(0)} = B(z), \quad (30)$$

$$\mathbf{M}_{21}^{(0)} = \frac{\mathbf{M}_{12}^{(0)}}{2}, \quad \mathbf{M}_{22}^{(0)} = z\mathbf{M}_{12}^{(0)}, \quad \mathbf{M}_{23}^{(0)} = z\mathbf{M}_{13}^{(0)}, \quad (31)$$

$$\mathbf{M}_{31}^{(0)} = (\gamma - 1)\mathbf{M}_{13}^{(0)}, \quad \mathbf{M}_{32}^{(0)} = 2(\gamma - 1)\mathbf{M}_{23}^{(0)}, \quad \mathbf{M}_{33}^{(0)} = (\gamma - 1)D(z) + \frac{3}{2}\left(\frac{5}{3} - \gamma\right)W(z), \quad (32)$$

$$\mathbf{M}_{12}^{(1)} = (1 - Z_0)\left(\frac{5}{3} - \gamma\right)B(z), \quad \mathbf{M}_{13}^{(1)} = \left(1 - \frac{f(\omega)}{\gamma}\right)C(z), \quad (33)$$

$$\mathbf{M}_{22}^{(1)} = z\mathbf{M}_{12}^{(1)}, \quad \mathbf{M}_{23}^{(1)} = z\mathbf{M}_{13}^{(1)}, \quad (34)$$

$$\mathbf{M}_{32}^{(1)} = (1 - Z_0)(\gamma - 1)\left(\frac{5}{3} - \gamma\right)E(z), \quad \mathbf{M}_{33}^{(1)} = \left(1 - \frac{f(\omega)}{\gamma}\right)\left[(\gamma - 1)F(z) + \frac{3}{2}\left(\frac{5}{3} - \gamma\right)A(z)\right], \quad (35)$$

with

$$f(\omega) = \frac{f_0}{1 - i\omega\tau_q} = f_0 \left(1 - \frac{i}{\mathcal{R}} \frac{f_0}{\gamma}\right)^{-1} \quad (36)$$

being the frequency-dependent Eucken factor. Moreover, we have introduced the abbreviations

$$A(z) = zW(z) + 1, \quad B(z) = \left(z^2 - \frac{1}{2}\right)W(z) + z, \quad C(z) = z\left(z^2 - \frac{3}{2}\right)W(z) + z^2 - 1, \quad (37)$$

$$D(z) = \left(z^4 - z^2 + \frac{5}{4}\right)W(z) + z^3 - \frac{z}{2}, \quad E(z) = \left(z^4 - z^2 - \frac{1}{4}\right)W(z) + z^3 - \frac{z}{2}, \quad (38)$$

$$F(z) = z\left(z^4 - 2z^2 + \frac{7}{4}\right)W(z) + z^4 - \frac{3}{2}z^2 + \frac{3}{2}. \quad (39)$$

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